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FOREWORD ON BEHALF OF THE EDITORS.

By E. R. HEDRICK, University of Missouri.

Collegiate Mathematics.—The lifework of hundreds of men is the teaching of collegiate mathematics; hundreds of other men are preparing for this same lifework. Thousands of students are now working and will work under the guidance of these men in collegiate courses in mathematics.

Suggestions of moment arise in the minds of many, yet there exist few means for transmitting an idea in this field to others. Any organized effort toward the dissemination of methods that are new, or of facts that are interesting, in this field is totally lacking.

These remarks apply, of course, only to the United States. Abroad, in every country in which mathematics holds a place commensurate with its position here, definite journals exist whose purpose is almost wholly to foster these interests. The AMERICAN MATHEMATICAL MONTHLY, beginning with this issue, proposes to afford an opportunity for any discussions that seem valuable upon collegiate mathematics, and the editors invite contributions concerning the methods of instruction as well as those that treat special topics or theorems.

Problems of Instruction.—Every teacher in this field has asked himself what topics he should best teach, why these topics are taught, how they should be approached, what purpose should be recognized as the main aim of collegiate mathematics. Ordinarily he has either found no answer whatever and has settled down to the basis of traditional procedure, or else he has satisfied himself perforce with partial answers that he could himself discover. Each man has been a power unto himself, but no man has received or given aid, at least to the extent of his ability.

Some conspicuous exceptions to this general state of affairs are mentioned below. What has been done is, however, sporadic, discontinuous, and of little influence,—certainly no such effort has been general, and the effects of what has been attempted are far from general.

The most general and most sustained work in this direction has recently culminated in the report of a committee of twenty mathematicians and engineers, under the chairmanship of Professor E. V. Huntington, of Harvard University.

This committee, was originally appointed at a joint meeting* of mathematicians and engineers, organized in 1907 by the Chicago Section of the American Mathematical Society; but it was authorized to report to the Society for the Promotion of Engineering Education, which body had already carried on a consistent series of discussions through a period of years on the teaching of mathematics to students of engineering; and these engineers, rather than the mathematicians, have furnished the medium, through their journal, for the publication of these discussions and of this final report.†

Though it may be contended that the aims of this Society are at least special, and touch only one side of our work, and though the spirit of many of the discussions may seem narrow to many, it is at least true that the attempt to suggest means for the improvement of mathematical teaching has been carried on consistently, courageously, and earnestly.

The efforts of this society, whose special aim is the improvement of engineering education, emphasize the variety of conditions that must be met by collegiate courses in mathematics at this time. The reasonable demands of the rapidly increasing schools of engineering, together with the normally increasing enrollments in academic classes, have vastly increased the problems that confront us.

Origin of New Problems.—The problems of to-day differ from those of a generation ago on account of three great movements that lie wholly outside the field of mathematics. Of these, the first is the spread and almost universal acceptance of the elective principle in colleges and in academic classes in institutions of every kind. To be sure, the tendency to-day seems to be to limit the scope of the elective system, but the days of the old fixed curriculum, with mathematics as a required study for all students, has passed, and forever.

Another movement of no less extent is the rapid increase in the number of those entering engineering and other semi-technical professions. In schools of engineering, a certain amount of mathematics is required of all students, though the amount varies. But this requirement that all students take collegiate courses in mathematics is not made, as was the older academic requirement, for the sake of general mental discipline and the training of the mind as a whole. While this motive may persist in part, the recent discussions by engineers, mentioned above, leave no room for doubt that the inherent reason, in this instance, is the belief in the direct usefulness to the engineer of the facts, the principles, and the processes to be learned in these courses.

The last, in point of time, of the three principal movements mentioned above is the present rapid change in educational theory, which is based to a large extent on experimental psychology, and which bears the same relation to the older pedagogy that chemistry bears to alchemy. Not all its problems are yet settled, many of them are now under intense discussion, but the manner of that discussion is the manner of science rather than the manner of controversy. One of its contributions that is widely accepted is that there are no such *fixed* faculties of the mind as those we had believed in rather implicitly; and that the training of such supposed faculties as the memory, for example, is impossible in its widest sense because there seems to be no one such faculty. The memory for number, for example, seems to be wholly unrelated to the memory for poetry, or the memory for forms.

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* A few printed copies of the papers read at this meeting may still be had from the secretary of the Chicago Section, H. E. Slaught, 58th Street and Ellis Ave., Chicago, Ill.

† This final report is in the form of a syllabus of courses in mathematics for students of engineering, and is for sale by the Society for the Promotion of Engineering Education, 43 E. Nineteenth St., New York.

Discipline.—To what extent training in any field carries over into any other field seems to be still under discussion. Whether the memory, for example, extends at all beyond the very region of the training received, whether a child who has been drilled in one special field gains in ability in another even closely allied field, seems yet not precisely known. That upon which there is complete agreement is that training of such a supposed faculty as the memory in *one* field does not strengthen the memory in *all* fields, but that it does strengthen the memory in at least some fields closely allied to that in which the training was received. The uncertainty that remains is to define how close the alliance must be.

While the question of training the memory is itself at least a minor consideration in the minds of many teachers of mathematics, the very analogous question of disciplinary training of the mind has been and is one of the strongest motives in the minds of many for the teaching of any course in mathematics. It was certainly the basis for the older requirement of mathematics in the early colleges. In so far as the modern theory of education renders uncertain the possibility of disciplinary training, it vitally affects every discussion of any mathematical topic. In how far training of the reasoning faculty may aid the student to better reasoning in general, in how far power to think in a particular field may be generally efficacious, are, in the opinion of many, problems whose solution lies in the future. We only know that the training received in reasoning, or thinking, upon one topic affects most strongly the ability to reason, or to think, on topics most closely associated with the topics in which the training was received. The uncertainty that is recognized is in describing accurately how close to the original topic a new one must be in order to show the effect of the previous training.

Selection of Topics.—Teachers of mathematics will not lightly abandon the value of the disciplinary training afforded by the subject, but the ability and scientific attitude of mind of teachers of collegiate mathematics, as compared with teachers in more elementary schools, will lead them to search for the truth about these questions and will make them cautious in basing their faith entirely upon an aim that is made questionable by these modern investigations in education.

One immediate result is the realization that the same amount of mental training, whatever that may be, seems likely to result from any one of several possible topics, if each of them is studied with the same thoroughness and attention. Does this make possible the selection of topics on grounds entirely different from mental discipline and, if the topics to be taught are selected for other reasons, will the resultant training be just as efficient?

This consideration, and the changed conditions which seem to raise the same demand, would point toward a very thorough and critical study of the motives, other than disciplinary training, for the study of mathematical courses, and of topics within those courses. Without discarding discipline as one of the desirable results of a mathematical training, a knowledge of other motives for the study of any one topic will serve only to enhance its value.

Comparative merits of different topics and of different methods of instruction can be realized most effectively if these other motives are considered, and if the disciplinary factors are temporarily ignored, for the claims of disciplinary value, as between two topics, are at best conflicting and uncertain, and the wholesome effect of any topic that is studied intensively will be recognized by all.

Interest.—The qualification that a study of any topic, in order to be effective, must be intensive and earnest will be insisted upon by all. It would seem therefore that profitable discussion may well take place concerning the means by which such work can be obtained. To arouse the student to his maximum effort, to stimulate his greatest efficiency, to gain the fullest measure of disciplinary good, there may be many plans of merit; but it is certain that all of them will have in common the holding of the student's attention, without which no effective study is thinkable. Any process for holding the attention in a sustained and serious fashion is described by saying that the student has become "interested" in the study or that his "interest" has been secured. But the word interest has been misused. The doctrine of interest has led to a mistaken doctrine of "amusement," than which no interpretation of the doctrine of interest could be more absurdly irrelevant. It is scarcely to be wondered at that the whole doctrine, sound as its real basis may be, has been misunderstood, abused, ridiculed, and that it appears to many as the antithesis of that very doctrine of discipline of which it is the sole effective support.

The Province of the Monthly.—This article is not itself a pedagogical document. What is here stated is perfectly well known in pedagogical circles, and we trust that we have not through ignorance, as we have certainly not through intention, overstepped the bounds of that which is generally recognized as finally established in the theory of education.

It is not the province of the MONTHLY to enter the field of general pedagogy, nor will the MONTHLY entertain discussions that are concerned with research in general pedagogy, or that deal with new theories of pedagogy, however important these contributions may seem. Journals for the publication of such contributions abound, and any article of worth in the general field is already more than liberally provided with possible means of publicity. In such matters, we should not venture to pose as expert critics of new matter, nor to pass upon the merits of articles. We must accept, as from a foreign field, the results of general pedagogy and of experimental psychology, and in order to be secure, we must remain always a little behind, rather than in advance of the times, on pedagogical theory.

What we do desire is to inspire, not a discussion along these lines, but rather a discussion of definite *mathematical* problems. It is our hope that this article will spur mathematicians to the realization of the new conditions that confront us, to the variety of problems that demand discussion, to the vital importance which their solution holds to a great body of intelligent people. For the discussion from this standpoint of mathematical questions, properly speaking, the MONTHLY will open its columns without reserve, except for those usual discretionary and advisory powers that are traditional in any editorship.

The Future.—This is not the last word that the MONTHLY will have to say upon this question. We shall indulge in rather little general pedagogical preaching, for we claim no professional attitude toward that subject. What seemed necessary has been said above. For this issue this is enough. We shall, however, not place any limitation upon the scope of our suggestions to readers and to possible contributors; and we shall hope, by these suggestions, to point out very explicit problems of a mathematical nature which might well be treated. We shall aim to awaken an interest in their discussion, even among those who now regard their own profession—the teaching of collegiate mathematics—with distrust as a possible field for that type of human thinking that is known as scientific research. In this distrust, we, the editors of the MONTHLY, emphatically announce that we do not share.

HISTORY OF THE EXPONENTIAL AND LOGARITHMIC CONCEPTS.

By FLORIAN CAJORI, Colorado College, Colorado Springs.

I. FROM NAPIER TO LEIBNIZ AND JOHN BERNOULLI I.
1614–1712.

LOGARITHMS OF POSITIVE NUMBERS.

The seventeenth and eighteenth centuries are notable in the history of algebra for important developments of the notations and ideas which mark its modern treatment. The study of this period in algebra is more than mere pastime. In the words of Felix Klein: "If we really desire to advance to a full understanding of the theory of logarithms, it is best to follow in broad outline the history of its creation."[1]

Logarithms were invented before our modern exponential notation, a^n, was introduced into algebra. To be sure, the use of algebraic symbols that were more or less different from the modern symbols to indicate powers and roots of a number had been suggested before the advent of the logarithm, but we shall

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[1] F. Klein, *Elementarmathematik vom höh. Standpunkte aus*, I, Leipzig, 1908, p. 325.